

Influence of quantum degeneracy on the performance of a Stirling refrigerator working with an ideal Fermi gas

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Abstract

The influence of quantum degeneracy on the performance of a Stirling refrigeration cycle is investigated, based on the equation of state of an ideal Fermi gas. The inherent regenerative losses and the coefficient of performance (COP) of the cycle are calculated. It is found that, under the condition of strong gas degeneracy, the COP of the cycle in the first approximation is a function only of the temperatures of the heat reservoirs, while under other conditions, the COPs of the cycle depend on the temperatures of the heat reservoirs and other parameters of the cycle. The results obtained here reveal the general performance characteristics of a Stirling refrigeration cycle having a Fermi gas as its working substance.

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Keywords: Stirling refrigeration cycle; Fermi gas; Quantum degeneracy; Regenerative loss; Coefficient of performance

1. Introduction

Recently, quantum-mechanical cycles have become interesting research subjects. The performances of quantum heat engines and refrigerators have been discussed [1–7]. Like classical cycles, quantum cycles may have different working substances.

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Nomenclature

C_V	Heat capacity at constant volume (J)
$F(z)$	Correction factor
$f(z)$	Fermi function
g	Weight factor
h	Planck's constant (Js)
k	Boltzmann's constant (J/K)
m	Rest mass of an ideal Fermi particle (kg)
N	Total number of particles of an ideal Fermi gas
n	Number density of of an ideal Fermi gas (m^{-3})
P	Pressure (Pa)
Q_{bc}	Amount of heat exchange between the working substance and the regenerator for a constant V_L process (J)
Q_{da}	Amount of heat exchange between the working substance and the regenerator for a constant V_H process (J)
Q_H	Amount of heat exchange between the working substance and the hot reservoir for a the high temperature process (J)
Q_L	Amount of heat exchange between the working substance and the cold reservoir for a low temperature process (J)
Q^V	Amount of heat exchange for a isochoric process (J)
r_v	Volume ratio of two isochoric processes
S	Entropy of the gas (J/K)
T	Temperature of the gas (K)
T_F	Fermi temperature (K)
T_H	Temperature of the hot reservoir (K)
T_L	Temperature of the cold reservoir (K)
U	Internal energy (J)
V	Volume (m^3)
V_H	Maximum volume of the gas (m^3)
V_L	Minimum volume of the gas (m^3)
v	Average volume that an ideal Fermi particle occupies (m^3)
v_H	Maximum average volume that an ideal Fermi particle occupies (m^3)
v_L	Minimum average volume that an ideal Fermi particle occupies (m^3)
W	Work input per cycle (J)
z	Fugacity of the gas
ΔQ	Regenerative loss (J)
ε	Coefficient of performance
ε_c	Coefficient of performance of a Carnot refrigeration-cycle
λ	Mean thermal wavelength (m)
μ	Chemical potential of the gas (J)

For example, the working substance in quantum cycles may be harmonic oscillator systems, spin systems, ideal quantum gases, and so on. Thus, it is necessary to study the influence of a specific working substance on the performance of a quantum cycle.

According to the theory of classical thermodynamics, a Stirling refrigeration cycle working with an ideal classical gas [8,9], a homogeneous ferromagnetic material [10,11], or a homogeneous ferroelectric material [12] may possess the condition of perfect regeneration through the use of a reversible regenerator. Its coefficient of performance (COP) is equal to that of a reversible Carnot refrigeration cycle. However, when the working substance is an ideal quantum gas, this conclusion is not valid because the quantum degeneracy of the gas particles becomes important [13–15]. Therefore, it is of significance to analyze how the performance of the Stirling refrigeration cycle working with an ideal quantum gas is affected by the quantum degeneracy of the working substances.

In the present paper, based on the equation of state of an ideal Fermi gas, the inherent regenerative losses and the COP of a Stirling regeneration cycle using an ideal Fermi gas as the working substance are analyzed. Under the conditions of quantum degeneracy, the ratio of the COP of the cycle to that of a reversible Carnot refrigeration cycle versus the temperature ratio of two heat reservoirs and the volume ratio of two isochoric processes are examined.

2. Thermodynamic properties of an ideal Fermi gas

Based on quantum statistics, the expressions of the pressure and the number density of an ideal Fermi gas are given by [13]

$$P = \frac{gkT}{\lambda^3} f_{5/2}(z) \quad (1)$$

and

$$n = \frac{N}{V} = \frac{1}{v} = \frac{g}{\lambda^3} f_{3/2}(z), \quad (2)$$

respectively, where g is a weight factor that arises from the “internal structure” of the particles (such as spin), $\lambda = h/(2\pi mkT)^{1/2}$ is the mean thermal wavelength of the particles, h is Planck’s constant, m is the rest mass of a particle, k is Boltzmann’s constant, T is the gas temperature, N is the total number of the particles, V is the volume of the system, v is the average volume that a Fermi particle occupies, $z = \exp(\mu/kT)$ is the fugacity of the gas, μ is the chemical potential of the gas,

$f_n(z) = \frac{1}{\Gamma(n)} \int_0^\infty \frac{x^{n-1} dx}{z^{-1} e^x + 1}$ is called the Fermi function, and $\Gamma(n)$ is the Gamma function.

By using Eqs. (1) and (2), the correction equation of state for an ideal Fermi

gas may be written as

$$P = nkTF(z), \quad (3)$$

where $F(z)$ is called the correction factor and it is defined as

$$F(z) = \frac{f_{5/2}(z)}{f_{3/2}(z)} \quad (4)$$

The internal energy and the entropy of an ideal Fermi gas are, respectively, given by [13]

$$U = \frac{3}{2} NkT \frac{f_{5/2}(z)}{f_{3/2}(z)} = \frac{3}{2} NkTF(z) \quad (5)$$

and

$$S = Nk \left[\frac{5f_{5/2}(z)}{2f_{3/2}(z)} - \ln z \right] = Nk \left[\frac{5}{2} F(z) - \ln z \right]. \quad (6)$$

From Eq. (5), one obtains the heat capacity at constant volume as

$$C_V = \left(\frac{\partial U}{\partial T} \right) = \frac{3}{2} Nk \frac{d}{dT} [TF(T, v)], \quad (7)$$

where $F(T, v)$ is a function of temperature T and volume v .

Using Eqs. (6) and (7), one can calculate the amounts of heat exchange in an isothermal and isochoric process. They are given by

$$Q_{if} = T(S_f - S_i) = NkT \left\{ \frac{5}{2} [F(z_f) - F(z_i)] - (\ln z_f - \ln z_i) \right\} \quad (8)$$

and

$$Q_{if}^v = \int C_V(T, v) dT = \frac{3}{2} Nk [T_f F(T_f, v) - T_i F(T_i, v)], \quad (9)$$

where the indices i and f refer to the initial and final states, respectively.

With the help of Eqs. (8) and (9), we can analyze the inherent regenerative losses and the COP of a Stirling refrigeration cycle working with an ideal Fermi gas.

3. A Fermi-Stirling refrigeration cycle

A Stirling refrigeration cycle using an ideal Fermi gas as the working substance is called a Fermi Stirling refrigeration cycle, which is composed of two isothermal and two isochoric processes. The entropy-temperature diagram of the cycle is shown in

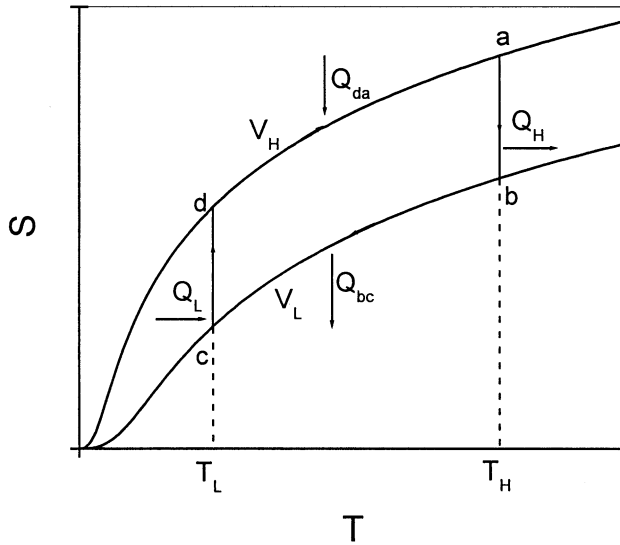


Fig. 1. The entropy-temperature diagram of a Fermi Stirling refrigeration cycle.

Fig. 1, where Q_L and Q_H are the amounts of heat exchange between the working substance and the heat reservoirs at temperature T_L and T_H during the two isothermal processes, Q_{bc} and Q_{da} are the amounts of heat exchange between the working substance and the regenerator during the two isochoric processes, and V_H and V_L are the maximum and minimum volumes of the gas system. All heats Q_L , Q_H , Q_{bc} , and Q_{da} are positive. In order to improve the performance of the cycle, a regenerator is often used in two isochoric processes.

By using Eqs. (8) and (9), the amounts of heat exchange in the various processes of the Stirling refrigeration cycle may be expressed as

$$Q_L = \frac{5}{2} NkT_L[F(z_d) - F(z_c)] - NkT_L(\ln z_d - \ln z_c), \quad (10)$$

$$Q_H = \frac{5}{2} NkT_H[F(z_a) - F(z_b)] - NkT_H(\ln z_a - \ln z_b), \quad (11)$$

$$Q_{da} = \int_d^a C_V(T, v_H) dT = \frac{3}{2} Nk[T_H F(z_a) - T_L F(z_d)], \quad (12)$$

and

$$Q_{bc} = \int_c^b C_V(T, v_L) dT = \frac{3}{2} Nk[T_H F(z_b) - T_L F(z_c)], \quad (13)$$

respectively, where z_a , z_b , z_c , and z_d are the fugacity of the gas in the a, b, c, and d states of Fig. 1.

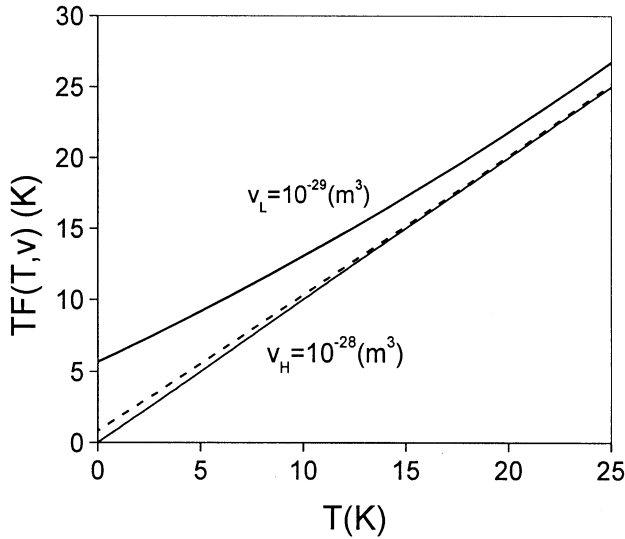


Fig. 2. The function $TF(T, v)$ versus the temperature curves for two different volumes. The solid curve is obtained for a constant minimum average volume, $v_L = 10^{-29} \text{ m}^3$, while the dashed curve is obtained for a constant maximum average volume, $v_H = 10^{-28} \text{ m}^3$.

From Eqs. (10)–(13), we obtain the work input per cycle as

$$\begin{aligned}
 W &= Q_H - Q_L + Q_{bc} - Q_{da} \\
 &= NkT_H[F(z_a) - F(z_b) + \ln z_b - \ln z_a] + NkT_L[F(z_c) - F(z_d) + \ln z_d - \ln z_c]
 \end{aligned} \tag{14}$$

The curves of the function $TF(T, v)$ varying with the temperature for two different volumes are shown in Fig. 2. Using Eq. (7) and comparing the slopes of the curves in Fig. 2, we can conclude that

$$C_V(T, v_H) > C_V(T, v_L) \tag{15}$$

and

$$\begin{aligned}
 \Delta Q &= Q_{bc} - Q_{da} \\
 &= \frac{3}{2} Nk \{ T_H [F(z_b) - F(z_a)] + T_L [F(z_d) - F(z_c)] \} < 0,
 \end{aligned} \tag{16}$$

where v_H and v_L are the maximum and minimum average volumes that an ideal Fermi particle occupies. Eq. (16) shows that the amount of heat exchange Q_{bc}

transferred into the regenerator in one isochoric process is smaller than that of heat exchange Q_{da} transferred from the regenerator in the other isochoric process. The inadequate heat in the regenerator per cycle can only be compensated from the hot reservoir in a timely manner, while refrigeration heat Q_L is unvarying. If not, the temperature of the regenerator would be changed such that the regenerator would not be operated normally. From Eqs. (10) and (14), the COP of the cycle can be expressed as

$$\varepsilon = \frac{Q_L}{W} = \frac{\frac{5}{2} T_L [F(z_d) - F(z_c)] - T_L (\ln z_d - \ln z_c)}{T_H [F(z_a) - F(z_b) + \ln z_b - \ln z_a] + T_L [F(z_c) - F(z_d) + \ln z_d - \ln z_c]}. \quad (17)$$

By using Eqs. (16) and (17), the non-perfect regeneration characteristics and the COP of the cycle may be analyzed in detail.

4. Three special cases

(1) Under the condition of strong gas degeneracy and in the first approximation, we can derive the fugacity and the correction factor (a detailed derivation is given in Appendix A) as

$$\ln z = \frac{T_F(v)}{T} - \frac{\pi^2}{12} \frac{T}{T_F(v)} \quad (18)$$

and

$$F(T, v) = \frac{2T_F(v)}{5T} + \frac{\pi^2 T}{6T_F(v)}, \quad (19)$$

where $T_F(v) = D/v^{2/3}$ is the Fermi temperature and $D = (3h^3/\pi)^{2/3}/(8 \text{ km})$.

By substituting Eqs. (18) and (19) into Eqs. (16) and (17), the inherent regenerative loss and the COP of the cycle in the first approximation can be expressed as

$$\Delta Q = \frac{\pi^2}{4D} Nk (T_H^2 - T_L^2) (v_L^{2/3} - v_H^{2/3}) < 0 \quad (20)$$

and

$$\varepsilon = \frac{T_L}{T_H - T_L} \frac{2T_L}{T_H + T_L} < \varepsilon_c, \quad (21)$$

respectively, where $\varepsilon_c = T_L/(T_H - T_L)$ is the COP of a reversible Carnot refrigeration

cycle. In such a case, the COP is only a function of temperature and independent of any other parameters.

(2) Under the condition of weak gas degeneracy, to a first approximation, we can derive the fugacity and the correction factor (a detailed derivation is given in Appendix B) as

$$z = 4\sqrt{2}E/(T^{3/2}v)[1 + 2E/(T^{3/2}v)] \quad (22)$$

and

$$F(T, v) = 1 + E \frac{1}{T^{3/2}v}. \quad (23)$$

where $E = h^3/32(\pi mk)^{3/2}$.

By substituting Eqs. (22) and (23) into Eqs. (16) and (17), the inherent regenerative loss and the COP of the cycle can be determined by

$$\Delta Q = \frac{3}{2}NkE(v_L^{-1} - v_H^{-1})\left(1/T_H^{1/2} - 1/T_L^{1/2}\right) < 0 \quad (24)$$

and

$$\varepsilon = \frac{T_L \ln(v_H/v_L) - (1/2)\left(E/T_L^{1/2}\right)(1/v_L - 1/v_H)}{(T_H - T_L)\ln(v_H/v_L) + E(1/v_L - 1/v_H)\left[\left(1/T_H^{1/2}\right) - \left(1/T_L^{1/2}\right)\right]}. \quad (25)$$

When ^3He gas is chosen as the ideal Fermi gas and Eq. (25) is used, one can generate the curves of the ratio of the COPs of the cycles, $\varepsilon/\varepsilon_c$, varying with the temperature ratio, $\tau = T_H/T_L$, of two heat reservoirs for three different volume ratios, $r_v = v_H/v_L$, of two isochoric processes, as shown in Fig. 3, where the minimum average volume that a particle occupies and the highest temperature are chosen to be $v_L = 10^{-29}\text{m}^3$ and $T_H = 177\text{ K}$, respectively. For a constant maximum average volume value, $v_H = 10^{-28}\text{m}^3$, or a constant minimum average volume value, $v_L = 10^{-29}\text{m}^3$, Fig. 4 gives the curves of the ratio of the COPs of the cycles varying with the volume ratio of two isochoric processes, where the highest and lowest temperatures are given as $T_H = 177\text{ K}$ and $T_L = T_F(v_L) = 16.6\text{ K}$, respectively. For a constant v_L , the coefficient of performance increases with an increase of r_v , while for a constant v_H , the coefficient of performance decreases with an increase of r_v . This implies that the COP of the cycle may be improved by increasing the maximum volume of the cycle.

When the temperature is high enough, $(1/2)\left(E/T_L^{1/2}\right)(1/v_L - 1/v_H)/\ln(r_v) \ll 1$ and $E(1/v_L - 1/v_H)\left[\left(1/T_H^{1/2}\right) - \left(1/T_L^{1/2}\right)\right]/\ln(r_v) \ll 1$. Eq. (25) is simplified as

$$\varepsilon = \frac{T_L}{T_H - T_L} = \varepsilon_c. \quad (26)$$

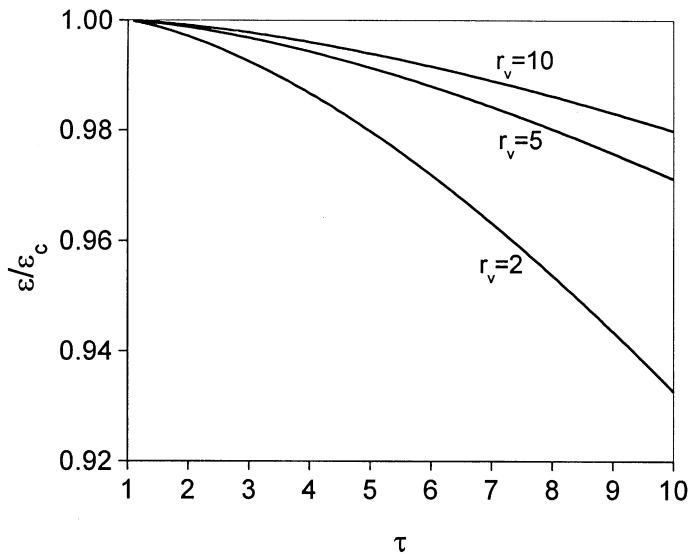


Fig. 3. The ratio of the COP of a Fermi Stirling refrigeration cycle to that of a reversible Carnot refrigeration cycle versus the temperature ratio curves for three different volume ratios of two isochoric processes. The curves are presented for ^3He . The minimum average volume and the highest temperature are given as $v_L = 10^{-29}\text{m}^3$ and $T_H = 177\text{K}$, respectively.

In this case, an ideal Fermi gas becomes an ideal classical gas and the working gas is always in the classical gas state throughout the cycle. The Stirling refrigeration cycle utilizing an ideal classical working gas may possess the condition of perfect regeneration, so that its COP is equal to that of a reversible Carnot refrigeration cycle.

(3) Under the condition of $T_H \gg T_F(v_L)$ and $T_L \ll T_F(v_H)$, the fugacity and correction factor may be expressed as

$$z(T_H, v) = 4\sqrt{2}E/\left(T_H^{3/2}v\right)\left[1 + 2E/\left(T_H^{3/2}v\right)\right], \quad (27)$$

$$F(T_H, v) = 1 + E/\left(T_H^{3/2}v\right), \quad (28)$$

$$\ln z(T_L, v) = \frac{T_F(v)}{T_L} - \frac{\pi^2}{12} \frac{T_L}{T_F(v)}, \quad (29)$$

and

$$F(T_L, v) = \frac{2T_F(v)}{5T_L} + \frac{\pi^2 T_L}{6T_F(v)}, \quad (30)$$

respectively. By substituting Eqs. (27)–(30) into Eqs. (16) and (17), the inherent regenerative loss and the COP of the cycle are determined by

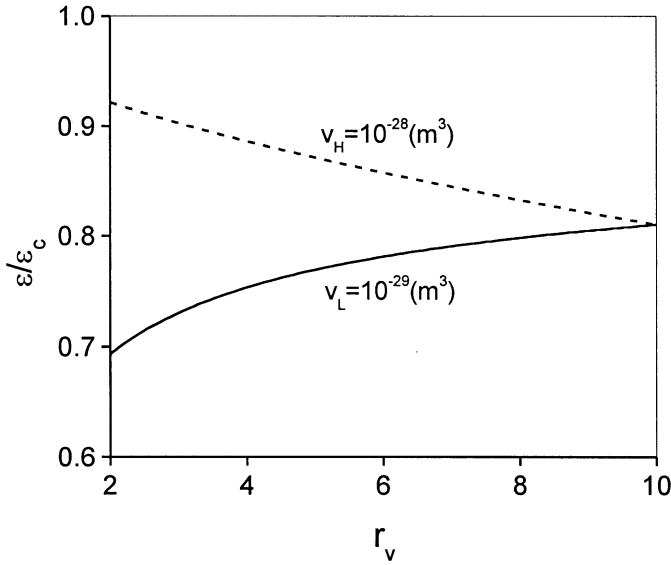


Fig. 4. The ratio of the COP of a Fermi Stirling refrigeration cycle to that of a reversible Carnot refrigeration cycle versus the volume ratio curves for ^3He . The solid curve is obtained for a constant minimum average volume, $v_L = 10^{-29}\text{m}^3$, while the dashed curve is obtained for the constant maximum average volume, $v_H = 10^{-28}\text{m}^3$. The highest and lowest temperatures are given as $T_H = 177\text{ K}$ and $T_L = 16.6\text{ K}$, respectively.

$$\Delta Q = \frac{3}{5} NkD \left(v_H^{-2/3} - v_L^{-2/3} \right) < 0 \quad (31)$$

and

$$\varepsilon = \frac{(\pi^2/2D)T_L^2(v_H^{2/3} - v_L^{2/3})}{T_H \ln(v_H/v_L) - (3D/5)(v_L^{-2/3} - v_H^{-2/3})}. \quad (32)$$

From Eq. (32) we can plot the ratio of the COPs versus the temperature ratio curves for three different volume ratios of two isochoric processes, as shown in Fig. 5, where the maximum average volume and the lowest temperature are given as $v_H = 10^{-28}\text{m}^3$ and $T_L = 1\text{ K}$, respectively. For a constant maximum average volume, $v_H = 10^{-28}\text{m}^3$, or a constant minimum average volume, $v_L = 10^{-29}\text{m}^3$, the ratio of the COPs versus the volume ratio curves are shown in Fig. 6, where the highest and lowest temperatures are given as $T_H = 177\text{ K}$ and $T_L = 1\text{ K}$, respectively. It is clearly seen from Fig. 6 that for a constant v_L , the coefficient of performance increases with an increase of r_v , while for a constant v_H , the COP decreases with an increase of r_v . These characteristics are similar to those in Fig. 4.

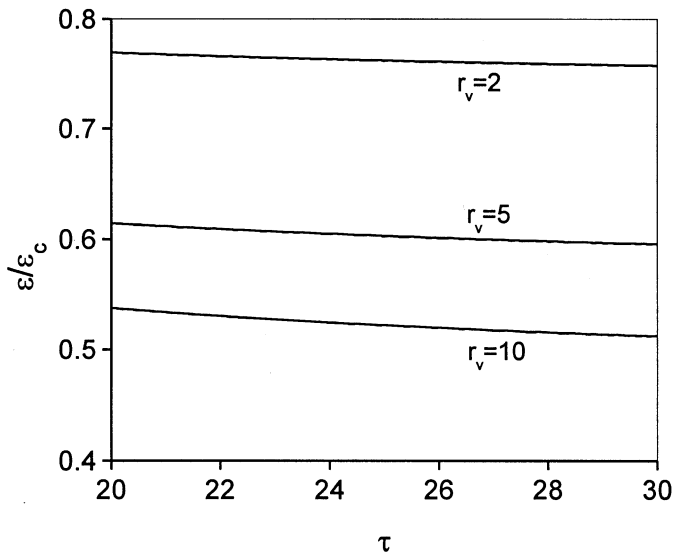


Fig. 5. The ratio of the COP of a Fermi Stirling refrigeration cycle to that of a reversible Carnot refrigeration cycle versus the temperature ratio curves for three different volume ratios of two isochoric processes. The curves are presented for ^3He . The maximum average volume and lowest temperature are given as $v_H = 10^{-28}\text{m}^3$ and $T_L = 1\text{ K}$, respectively.

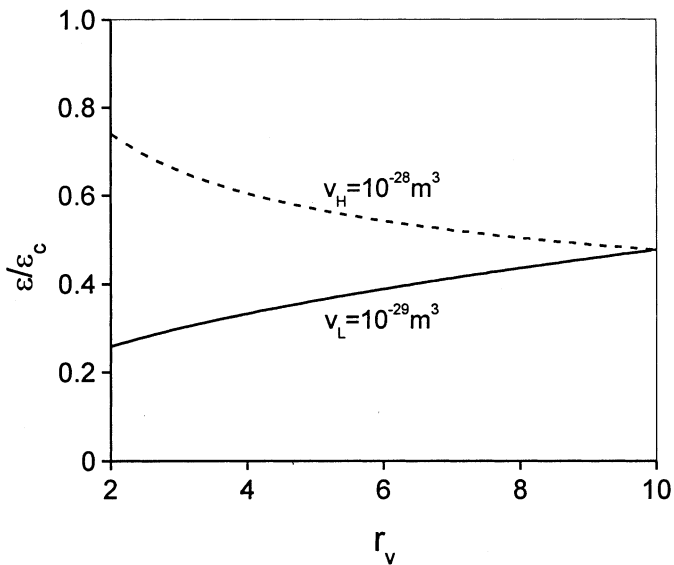


Fig. 6. The ratio of the COP of a Fermi Stirling refrigeration cycle to that of a reversible Carnot refrigeration cycle versus the volume ratio curves for ^3He . The solid curve is obtained for a constant minimum volume, $v_L = 10^{-29}\text{m}^3$, while the dashed curve is obtained for a constant maximum average volume $v_H = 10^{-28}\text{m}^3$. The highest and lowest temperatures are given as $T_H = 177\text{ K}$ and $T_L = 1\text{ K}$, respectively.

5. Conclusions

A Stirling refrigeration cycle working with an ideal Fermi gas does not possess the condition of perfect regeneration due to the quantum degeneracy of the Fermi gas. The COP of the cycle is smaller than that of a classical Stirling refrigeration cycle, whose COP is equal to that of a reversible Carnot refrigerator through the use of a reversible regenerator. Under the condition of strong gas-degeneracy, the COP of a Fermi Stirling refrigeration cycle, to a first approximation only depends on the temperatures of the heat reservoirs. Under the condition of weak gas degeneracy, the COP of the cycle is not only a function of temperatures but also dependent on the volumes and other parameters.

Acknowledgements

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Appendix A

In low temperature and high density conditions, i.e. under the condition of strong gas-degeneracy, the Fermi function $f_n(z)$ can be expressed as asymptotic expansions in powers of the quantity $(\ln z)^{-1}$ [13], i.e.

$$f_{5/2}(z) = \frac{8}{15\sqrt{\pi}} (\ln z)^{5/2} \left[1 + \frac{5\pi^2}{8} (\ln z)^{-2} + \dots \right], \quad (\text{A1})$$

$$f_{3/2}(z) = \frac{4}{3\sqrt{\pi}} (\ln z)^{3/2} \left[1 + \frac{\pi^2}{8} (\ln z)^{-2} + \dots \right]. \quad (\text{A2})$$

Using Eqs. (A1) and (A2), we can obtain the expression of the correction factor as

$$\begin{aligned} F(z) &= f_{5/2}(z)/f_{3/2}(z) = \frac{2}{5} \ln(z) \frac{1 + \frac{5}{8} \pi^2 (\ln z)^{-2} + \dots}{1 + \frac{1}{8} \pi^2 (\ln z)^{-2} + \dots} \\ &= \frac{2}{5} \ln(z) \left[1 + \frac{\pi^2}{2} (\ln z)^{-2} + \dots \right] \end{aligned} \quad (\text{A3})$$

To a first approximation [13],

$$\ln z = \frac{\varepsilon_F}{kT} \left[1 - \frac{\pi^2}{12} \left(\frac{kT}{\varepsilon_F} \right)^2 \right], \quad (\text{A4})$$

where $\mu_F = \left(\frac{3n}{4\pi g} \right)^{2/3} \left(\frac{h^2}{2m} \right)$ is the Fermi energy. Substituting Eq. (A4) into (A3), to a first approximation we can obtain

$$F(T, v) = \frac{2T_F}{5T} + \frac{\pi^2 T}{6T_F}, \quad (\text{A5})$$

where $T_F(v) = D/v^{2/3}$ is the Fermi temperature.

Appendix B

In high temperature and low-density conditions, i.e. under the condition of weak gas degeneracy, the Fermi function $f_n(z)$ can be expressed as an asymptotic expansion in powers of the quantity (z) [13],

$$f_{5/2}(z) = z - \frac{z^2}{2^{5/2}} + \frac{z^3}{3^{5/2}} - \dots, \quad (\text{B1})$$

$$f_{3/2}(z) = z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} - \dots. \quad (\text{B2})$$

Using Eq. (B1) and (B2), we can obtain the expression of the correction factor as

$$F(z) = \frac{f_{5/2}(z)}{f_{3/2}(z)} = 1 + \frac{1}{4\sqrt{2}} z + \dots. \quad (\text{B3})$$

Solving Eqs. (2) and (B2), we can obtain

$$y = f_{3/2}(z) = z - z^2/2^{3/2} + z^3/3^{3/2} - \dots, \quad (\text{B4})$$

where

$$y = \frac{\lambda^3}{gv} \quad (\text{B5})$$

and

$$z = b_1 y + b_2 y^2 + \dots. \quad (\text{B6})$$

Substituting Eq. (B6) into Eq. (B4) and comparing the coefficients of the same powers of y , we can obtain $b_1 = 1$, $b_2 = 1/2^{3/2}$, $\dots$. Consequently, one can obtain

$$z = \frac{\lambda^3}{gv} + \frac{1}{2^{3/2}} \left(\frac{\lambda^3}{gv} \right)^2 + \dots \quad (\text{B7})$$

Substituting Eq. (B7) into Eq. (B3), to a first approximation we can obtain the expression for the correction factor as

$$F(T, v) = 1 + E/(T^{3/2}v). \quad (\text{B8})$$

In the first approximation, Eq. (B7) can be written as

$$z(T, v) = 4\sqrt{2}E/(T^{3/2}v)[1 + 2E/(T^{3/2}v)]. \quad (\text{B9})$$

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